The structure of Spacetime

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Abstract — The special theory of relativity postulates that the speed of light is always constant in a vacuum. However, recent experiments [5, 6, 7] indicate that the photon has a very small but nonzero mass, which will affect the speed of light. According to quantum theory, the photon is a massless particle, and it is purely energy. But the photon having mass will contradict the photon's definition in quantum theory, because in the special theory of relativity, there is a relationship between mass and energy. In this paper, I will discuss the meaning of the photon's mass and how we can unify the definitions of the photon in quantum theory (pure energy) and in special relativity (a particle). At first, I will discuss the relativistic mass and then derive new relativistic equations regarding time and length. As we will see, these new equations are useful for the interpretation of the truth, which says: Why can only massless particles spread over an infinite range? However, we have to introduce a new model using these new equations. Finally, we will show that the origin of the elementary particles' masses can be interpreted in this model.

Index Terms—Spacetime, photon mass, Lorentz transformations, relativistic mass.

1 Introduction

In 1905, Einstein published three important papers, one of them under title "Zur Elektrodynamik bewegter Körper", or "On the Electrodynamics of Moving Bodies", which is today known as the special theory of relativity. He started from two postulates: one states that the laws of physics are the same in all inertial frames of reference and the other states that the speed of light in a vacuum is the same in all inertial frames of reference and that it is also independent of the motion of the source. From these postulates, he derived very important results: (i) observers measuring any clock will observe it running slower if it moves relative to them, (ii) observers measuring any length will observe it contract if it moves relative to them and (iii) Newton's second law and the equations of momentum and kinetic energy must be modified to be consistent with the principle of special relativity and Lorentz transformations.

The equations in special relativity always include a factor, called the Lorentz factor. It is given by:

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

where u is the speed of the particle and c is the speed of light. There is no particle that can reach the speed of light because, in this case, the Lorentz factor goes to infinity. In relativity, the equations of time intervals, lengths and energy are given by [1]:

$$\Delta t' = \gamma \Delta t$$

$$x' = \frac{x}{\gamma}$$

$$E = \gamma m c^2$$

When $u = c$, the relativities of time, total energy or the mass* go to infinity, and the relativity of length goes to zero. Now, if the Lorentz factor has been reversed in each of these equations, we will find that, in the relativity of time, the mass goes to zero and the relativity of length goes to infinity. These results are useful for interpreting the photon's behaviour. The photon is considered to be a massless particle, and it is spread over an infinite range. If we take the equations of the inverse relativity of length and energy, we will find that, when the particle can reach the speed of light (this topic is discussed later), its length will be infinity and its mass will be zero. Thus, we know that the mass of a photon is zero and that its range is infinity.

In this paper, I will derive these inverse equations of relativity and the new Lorentz transformations, which come from these new inverse equations. Also, I will discuss the relation between the origin of particle's masses and the contraction – extension of spacetime.

1 Deriving the relativity of time intervals

The time it takes light to make one full trip inside the frame of reference S', which has velocity u relative to frame of reference S, according to figure (1-1), is [1]:

$$\Delta t = \frac{d}{c} \qquad (1)$$

Relative to reference frame S', the time is [4]:

$$\Delta t' = \frac{L}{c} = \frac{\sqrt{d^2 + (u\Delta t')^2}}{c} \qquad (2)$$

$$\Delta t' = \sqrt{\frac{d^2}{c^2 - u^2}} \Rightarrow$$

$$\Delta t' = \frac{d}{c}\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \qquad (3)$$

Or

$$\Delta t^{\prime} = \Delta t \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma \Delta t \qquad (4)$$

Now, it is necessary to find equation (2) from equation (4) because, later, I will use a similar method to find new equations. By squaring $\frac{d}{c}$ and then substituting the squared expression into equation (4), we obtain:

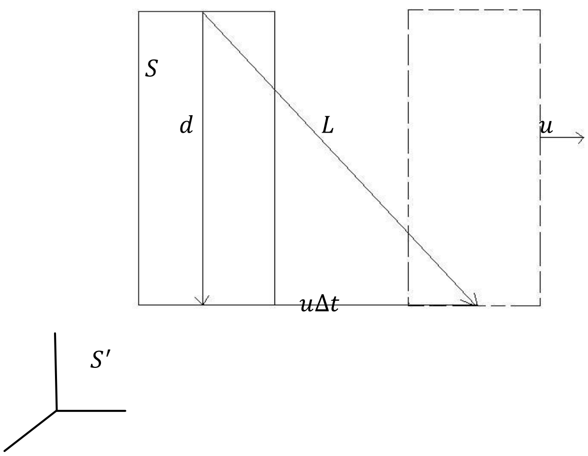

Figure (1-1) A frame of reference S moves with velocity u relative to rest frame of reference $S^{\prime}$.

$$\Delta t' = \sqrt{\frac{d^2}{c^2-u^2}}$$

Then, by squaring both sides and multiplying each one by c^2-u^2, we obtain:

$$\Delta t^{\prime} = \frac{\sqrt{d^2+(u\Delta t^{\prime})^2}}{c}$$

The result is equation (2).

2 Deriving the relativity of length

The time required for a particle of light to make one full round trip inside the reference frame S', which has velocity u relative to reference frame S, according to figure (1-2), is [4]:

$$\Delta t = \frac{2x}{c} \qquad (5)$$

The time required for the signal to reach the front end, relative to frame $S^{\prime}$, is [4]:

$$\Delta t_1{}^{\prime} = \frac{x^{\prime} + u\Delta t_1}{c} \Longrightarrow \Delta\ {}_1{}^{\prime} = \frac{x^{\prime}}{c-u} \qquad (6)$$

In addition, the return time is [4]:

$$\Delta t_2{}^{\prime} = \frac{x^{\prime} - u\Delta t_2}{c} \Longrightarrow \Delta t_2{}^{\prime} = \frac{x^{\prime}}{c+u} \qquad (7)$$

Thus, the total time is:

$$\Delta t^{\prime} = \Delta t_1{}^{\prime} + \Delta t_2{}^{\prime} = \frac{2x^{\prime}}{c\left(1-\frac{u^2}{c^2}\right)} \qquad (8)$$

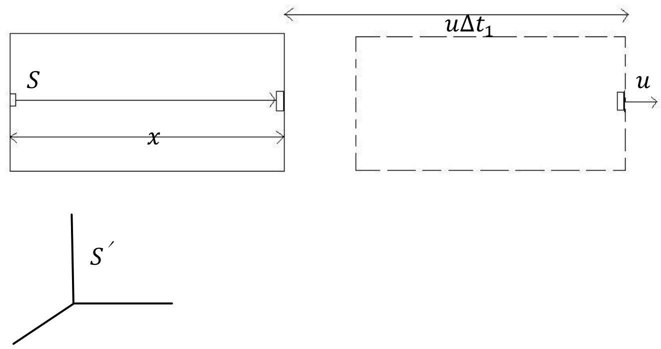

Figure (1-2) A frame of reference S moves with velocity u relative to the rest frame of reference $S^{\prime}$.

By substituting equations (8) and (5) into equation (4), we obtain:

$$x^{\prime} = \frac{x}{\gamma} \qquad (9)$$

Again, it is necessary to find equations (6) and (7) from equation (8). By multiplying both the numerator and denominator in equation (8) by c^2, we obtain:

$$\Delta t^{\prime} = \frac{2x^{\prime}}{(c-u)(c+u)} = \frac{x^{\prime}}{(c-u)} + \frac{x^{\prime}}{(c+u)}$$

Thus,

$$\Delta t_1{}^{\prime} = \frac{x^{\prime}}{c-u} = \frac{x^{\prime} + u\Delta t_1}{c} \quad ; \quad \Delta t_2{}^{\prime} = \frac{x^{\prime}}{c+u} = \frac{x^{\prime} - u\Delta t_2}{c}$$

3 The rest and relativistic mass

"What does determine the momentum and energy of massless particle? Not the mass (that's zero by assumption); not the speed (that's always c). However, then, how does a photon with an energy of 2 eV differ from a photon with an energy of 3 eV? Relativity offers no answer to this question, but curiously enough quantum mechanics does……… the 2 eV photon is red, and the 3 eV photon is purple!"

-David Griffiths-

In this section, I will discuss the concept of mass; in particular, I will address the definitions of the rest mass and the relativistic mass?

Modern physics assumes that the photon has no mass. However, according to recent experiments, the photon has a small but nonzero mass [5, 6, 7]. Thus, we have to reconsider our knowledge about the concept of mass.

The mass is defined in two different ways in the special theory of relativity. One approach defines mass as rest or invariant mass, and the other approach defines mass as relativistic mass. We can define the rest mass as an invariant quantity, which is the same for all reference frames, while the relativistic mass depends on the velocity of the reference frame. These definitions are general, but we have to take the their implications more seriously. All physicists likely accept the rest mass concept, but not all of them accept the relativistic mass concept [8, 9].

The relativistic mass is given by:

$$m(u) = \frac{m}{\sqrt{1 - \frac{u^2}{c^2}}} \qquad (10)$$

where m is the rest mass, which does not depend on the reference frame. We know that the mass and the energy are in fact different definitions of one substance [1]:

$$E = mc^2 \qquad (11)$$

where E is the rest energy and m is the invariant mass. This equation indicates that, if any particle has rest energy, it must also have invariant mass given by:

$$m = \frac{E}{c^2} \qquad (12)$$

But if the particle has velocity $(v \sim c)$, equation (11) will be:

$$E^2 = p^2c^2 + m^2c^4 \qquad (13)$$

When $v = 0$, we get back equation (11). We know that the momentum of any particle is given by:

$$p = mu \qquad (14)$$

Thus, from equation (12), the momentum becomes:

$$p = \frac{E}{c^2}u \qquad (15)$$

If the photon has no mass, equation (13) will be:

$$p = \frac{E}{c} \qquad (16)$$

By setting equations (15) and (16) equal, we obtain:

$$u = c$$

This result indicates that there is no rest frame for photons. They have no rest energy; instead, their total energy is purely kinetic. However, as stated above, according to recent experiments the photon has a small but nonzero mass. Thus, the first problem is the following: these photons would propagate at less than the speed of light according to special relativity, which states that the speed of light must be a constant in every frame of reference. Actually, this fact directly supports the idea that the mass depends on the velocity: when the photon has any nonzero mass, it would propagate at less than the speed of light.

Now let us consider both states of mass of the photon, i.e., its rest mass and its relativistic mass. First, if we consider the mass, which is measured experimentally, to be rest mass, we can state:

$$E = mc^2 = hf_\circ \implies \qquad (17)$$

$$m = \frac{hf_\circ}{c^2} \qquad (18)$$

In this case, I can only show equivalence between those two equations if $hf_\circ$ represents the rest energy. However, $f_\circ$ is the frequency, so we have to introduce the rest frequency concept in such a way that it is consistent with the rest energy definition. However, the concept of rest frequency is meaningless because rest frequency is the frequency of the photons when they are at rest. This idea is paradoxical because, as we conclude above, there is no rest frame of reference for photons. Thus, we have to equivalence between the relativistic energy formula and the photon quantum formula:

$$\frac{mc^2}{\sqrt{1 - \frac{u^2}{c^2}}} = hf \qquad (19)$$

Or

$$\lambda = \frac{h}{mc}\sqrt{1-\frac{u^2}{c^2}} \quad (20)$$

which is the de Broglie relation for the relativistic case. From equation (19):

$$m = \frac{hf}{c^2}\sqrt{1-\frac{u^2}{c^2}} \quad (21)$$

Now, because hf is the photon's energy and because we know that the rest mass is equal to its energy divided by the square of the speed of light, we can write equation (21) as:

$$m(u) = m\sqrt{1-\frac{u^2}{c^2}} \quad (22)$$

Equation (22) is an important formula and is another variation of the de Broglie relation. This equation must be true, but it must also have another meaning, as we will discuss later.

4 Conservation of mass-energy

In 1905, Einstein proposed a "thought experiment" [10] to develop his famous equation (11). Imagine that there is a cylinder of mass M and length L that is initially at rest, and then imagine that a photon of light is emitted from the left, as in figure (1-3). From equation (16), we know that the photon carries a momentum. From the conservation of energy, the cylinder must recoil to the left with speed v:

$$v = \frac{E}{Mc} \qquad (23)$$

Because the cylinder is very massive compared with the photon, the recoil speed will be small ($v \ll c$), and then the cylinder will move by a small amount, Δx, to the left:

$$\Delta x = v\Delta t = \frac{EL}{Mc^2} \qquad (24)$$

where Δt is the time it takes the light to reach the end of the box. When the light strikes the end of the box, it will transfer its momentum to the box, causing the box to stop. The centre of mass must appear to have been moved to the left, but the centre of mass cannot move because the box is an isolated system. To solve this problem, Einstein assumed that the photon carries mass m, so by requiring the centre of mass of the box to remain fixed, the following equation must be true:

$$mL = M\Delta x \qquad (25)$$

Solving for m and using equation (24) we obtain:

$$m = \frac{E}{c^2} \qquad (26)$$

This result is the same as equation (11). The conclusion of this experiment is that the photon has energy E and that it transfers mass, according to equation (26). In other words, the mass of the particle measures its energy content. In addition, if its energy varies with velocity, then the mass (the measured energy content) must also vary along with the velocity. Therefore, we have to distinguish between the rest mass and the relativistic mass.

If the mass is energy and the energy is different in different reference systems, the mass must also be different in different systems, although it is not the component of a four–vector $(E, \boldsymbol{p})$. Whatever could be said about the energy could just as well be said about $m(v)$. The conservation of energy is nothing more than the conservation of relativistic mass multiplied by a factor of c^2. Actually, we cannot escape from the relativistic mass definition at the theoretical level. However, at the experimental level, I feel that the definition remains a mystery.

5 Compton scattering with photon mass

There is another method to test the value of a photon's mass: finding the Compton scattering formula with this photon's mass. Consider the collision of one photon of mass m with one electron of mass m_{el}, which is assumed to be initially at rest in the laboratory coordinate system. Because of the conservation of energy, the difference in the photon's energy equals the kinetic energy that is gained by the electron:

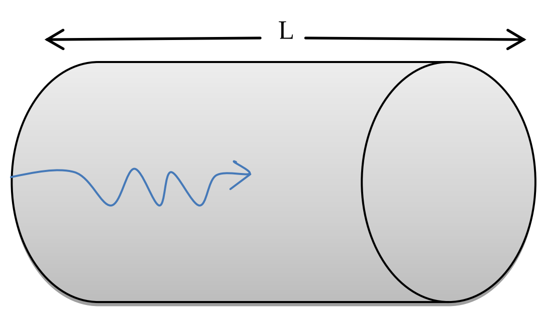

Figure (1-3) A photon is emitted at the left side of a cylinder, and it is absorbed at the right side. Because Einstein required that the centre of mass not move, he concluded that the photon has energy E, which transfers mass, according to equation (26).

$$hf - hf' = KE \qquad (27)$$

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where $f^{\prime}$ is the frequency of the photon after the collision. If we consider that the photon has no mass, then from equation (13):

$$p = \frac{hf}{c} \qquad (28)$$

However, if we take the photon's mass into account, the above equation will be:

$$p = \frac{hf}{c} - mc \qquad (29)$$

The photon's initial momentum is $\frac{hf}{c} - mc$, and the scattered photon's momentum is $\frac{hf^{\prime}}{c} - mc$. In addition, the initial and final electron momenta are 0 and p, respectively. Thus, we can write the following equation in the photon's original direction:

$$\frac{hf}{c} - mc + 0 = \left(\frac{hf^{\prime}}{c} - mc\right)\cos\phi + p \qquad (30)$$

And equation can also be expressed perpendicular to the photon's original direction:

$$0 = (\frac{hf^{\prime}}{c} - mc)\sin\phi - p\sin\theta \qquad (31)$$

The angle ϕ is the angle between the directions of the initial and scattered photons, and θ is the angle between the directions of the initial photon and the recoiled electron. By multiplying equations (30) and (31) by c, they can be rewritten as:

$$pc\cos\theta = hf - hf^{\prime}\cos\phi - mc^2(1-\cos\phi) \qquad (32)$$

$$pc\sin\theta = hf^{\prime}\sin\phi - mc^2\sin\phi \qquad (33)$$

Then, by squaring each of these equations and performing the necessary algebra, we obtain the following:

$$p^2c^2 = (hf)^2 - 2(hf)(hf^{\prime}\cos\phi) + \left(hf^{\prime}\right)^2 - 2mc^2(1 - \cos\phi)\left(h(f+f^{\prime}) - mc^2\right) \qquad (34)$$

Next, we equate between the equations for the total energy of a particle:

$$E = KE + m_{el}c^2 \qquad (35)$$

$$E = \sqrt{p^2c^2 + {m_{el}}^2c^4}$$

to obtain:

$$p^2c^2 = KE^2 + 2m_{el}c^2KE \qquad (36)$$

Next, substitute the value of KE from equation (27), and then equate the result with equation (34) to obtain the following:

$$2m_{el}c^2\left(hf - hf^{\prime}\right) = 2(hf)(hf^{\prime})(1-\cos\phi) - 2mc^2(1 - \cos\phi)\left(h(f+f^{\prime}) - mc^2\right) \qquad (37)$$

Finally, divide by $2h^2c^2$ to find:

$$\frac{m_{el}c}{h}\left(\frac{1}{\lambda} - \frac{1}{\lambda^{\prime}}\right) = \frac{1-\cos\phi}{\lambda\lambda^{\prime}} - \frac{mc}{h^2}(1-\cos\phi)\left(h\left(\frac{\lambda^{\prime}+\lambda}{\lambda\lambda^{\prime}}\right) - mc\right) \qquad (38)$$

Or

$$\lambda^{\prime} - \lambda = \frac{h}{m_{el}c}(1-\cos\phi)\left[1 - \frac{mc}{h^2}\left(h(\lambda^{\prime}+\lambda) - mc\lambda\lambda^{\prime}\right)\right] \qquad (39)$$

This equation is the Compton scattering with a photon having mass m. As we can see, when $m = 0$, the formula reverts back to the original version of Compton's equation. Because we can use various scattering angles to find the value of m, this is a valuable formula. Compton's formula depends on equation (13), and this formula originally came from the two equations of relativistic total energy and relativistic momentum. Thus, the value of the photon's mass from this equation also depends on the velocity, which provides additional support for the relativistic mass concept.

6 The extension concept of the relativity of time intervals and length

Equation (22) is similar to the original equation of relativistic mass (10), but the Lorentz factor has been inversed. If this equation is correct, it follows that the equations for the time interval and length must exist with an inverse Lorentz factor.

We can write equation (4) as:

$$d\tau = \sqrt{1 - \frac{u^2}{c^2}}\,dt \qquad (40)$$

where $d\tau$ is called proper time, or the time associated with the moving object, while dt is the time relative to the frame of reference outside of the moving object. Now, define the proper velocity 4-vector η^{μ}[11]:

$$\eta^{\mu} \equiv \frac{dx^{\mu}}{d\tau} \qquad (41)$$

The zeroth component of the above vector is [11]:

$$\eta^{0} = \frac{dx^{0}}{d\tau} = c\frac{dt}{d\tau} = \frac{c}{\sqrt{1-\frac{u^2}{c^2}}} \qquad (42)$$

Relativistic momentum is the spatial part of this 4-vector:

$$p^{\mu} \equiv m\eta^{\mu}$$

with

$$p^{0} = m\eta^{0} = \frac{mc}{\sqrt{1-\frac{u^2}{c^2}}} \qquad (43)$$

From this equation, Einstein called

$$m(u) \equiv \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}$$

the relativistic mass, but Einstein later wrote, "The idea of relativistic mass is “not good” because no clear definition can be given"[8]. However, as stated previously, we have to review the concept of relativistic mass because recent experiments indicate that the photon has a mass.

From equation (22), we can write the following equation after multiplying both sides by the velocity:

$$p = mu\sqrt{1-\frac{u^2}{c^2}} \qquad (44)$$

This equation implies that there is must be another relation between $d\tau$ and dt that is consistent with equation (44). This relation must take the form:

$$\frac{dt}{d\tau} = \sqrt{1-\frac{u^2}{c^2}} \qquad (45)$$

Thus, the zeroth component of equation (42), according to equation (45), must be:

$$\eta^{0'} = \frac{dx^{0}}{d\tau} = c\frac{dt}{d\tau} = c\sqrt{1-\frac{u^2}{c^2}} \qquad (46)$$

Then, by using (46), the zeroth component of (45) is:

$$p^{0'} = m\eta^{0'} = mc\sqrt{1-\frac{u^2}{c^2}} \qquad (47)$$

Thus, we have another important equation, i.e., equation (45). We can write it as:

$$\Delta t' = \frac{1}{\gamma}\Delta t \qquad (48)$$

If this equation is correct, there must exist another equation for the relativity of length that obeys equation (48). Because we know that the length contraction is consistent with the time dilation, the equation that is consistent with equation (48) must take the form:

$$x' = \gamma x \qquad (49)$$

These two equations predict that, although the relativity of time will decrease with a higher velocity, the relativity of length increases with a higher velocity. I will discuss this concept in more detail in section 8.

We derived equation (2) from equation (4). Here, we will use the same method to find the shape of equation (2) but from equation (48). Thus, by multiplying the numerator and denominator of equation (44) by c^2 and by using $\Delta t = \frac{d}{c}$, we obtain:

$$\Delta t' = \sqrt{\frac{d^2}{c^2} - \frac{d^2}{c^4}u^2} \qquad (50)$$

Squaring both sides and then multiplying each side by $(c^2 - u^2)$ yields:

$$c^2\Delta t'^{\,2} = d^2 - d^2\frac{u^2}{c^2} - u^2\frac{d^2}{c^2} + u^4\frac{d^2}{c^4} + \Delta t'^{\,2}u^2$$

$$\Delta t' = \frac{\sqrt{d^2\left(1-2\frac{u^2}{c^2}+\frac{u^4}{c^4}\right) + \Delta t'^{\,2}u^2}}{c} \qquad (51)$$

For the relativity of length, equation (8) must take the form:

$$\Delta t' = \frac{2x'}{c}\left(1-\frac{u^2}{c^2}\right) \qquad (52)$$

By combining equations (5) and (48), we obtain:

$$\Delta t' = \frac{2x}{c}\sqrt{1-\frac{u^2}{c^2}} \qquad (53)$$

Equating equations (52) and (53) yields:

$$x' = \gamma x$$

The result is equation (49).

7 Lorentz transformation in contracted and extended space-time

Suppose we have two frames of reference, S and $S^{\prime}$, and suppose $S^{\prime}$ moves with constant velocity u relative to the frame S. The distance from the origin of frame S to the point P in the frame $S^{\prime}$ is given according to the Lorentz transformation [3]:

$$x = ut + \frac{x^{\prime}}{\gamma} \qquad (54)$$

where x is the distance from the origin of frame S to some point P in the frame $S^{\prime}$ and $x^{\prime}$ is the distance from the origin of frame $S^{\prime}$ to the same point P. By solving equation (54) for $x^{\prime}$, we obtain:

$$x^{\prime} = (x - ut)\gamma \qquad (55)$$

We can write equation (50) as [3]:

$$x^{\prime} = -ut^{\prime} + \frac{x}{\gamma} \qquad (56)$$

By equating (55) and (56), we obtain:

$$t^{\prime} = \gamma\left(t - \frac{ux}{c^2}\right) \qquad (57)$$

This equation is the time transformation between reference frames S and $S^{\prime}$.

The Lorentz transformation must take a different shape, due to equations (48) and (49). Equation (54) must take another shape, and due to equation (49), it must be:

$$x = ut + x^{\prime}\gamma \qquad (58)$$

and

$$x^{\prime} = -ut^{\prime} + x\gamma \qquad (59)$$

By solving equation (58) for $x^{\prime}$ and then setting it equal to (59), we obtain:

$$t^{\prime} = \gamma\left(t + \frac{ux}{c^2} - \frac{u^2 t}{c^2}\right) \qquad (60)$$

This equation is the time transformation according to equations (48) and (49).

To find the relativistic velocity addition law, divide the differentiating of equation (57) by the differentiating of equation (54):

$$v_x^{\prime} = \frac{dx^{\prime}}{dt^{\prime}} = \frac{(dx - udt)\gamma}{\left(dt - \frac{u}{c^2}dx\right)\gamma} = \frac{\frac{dx}{dt} - u}{1 - \frac{u}{c^2}\frac{dx}{dt}} = \frac{v_x - u}{1 - \frac{uv_x}{c^2}} \qquad (61)$$

Similarly:

$$v_y^{\prime} = \frac{dy^{\prime}}{dt^{\prime}} = \frac{dy}{dt^{\prime}} = \frac{v_y}{1 - uv_x/c^2}\frac{1}{\gamma} \qquad (62)$$

$$v_z^{\prime} = \frac{dz^{\prime}}{dt^{\prime}} = \frac{dz}{dt^{\prime}} = \frac{v_z}{1 - \frac{uv_x}{c^2}}\frac{1}{\gamma} \qquad (63)$$

If we use the same method on equations (58) and (60), the results will be:

$$v_x^{\prime} = \frac{dx^{\prime}}{dt^{\prime}} = \frac{(v_x - u)}{\left(1 + \frac{uv_x}{c^2} - \frac{u^2}{c^2}\right)}\frac{1}{\gamma^2} \qquad (64)$$

$$v_y^{\prime} = \frac{dy^{\prime}}{dt^{\prime}} = \frac{dy}{dt^{\prime}} = \frac{v_y}{\left(1 + \frac{uv_x}{c^2} - \frac{u^2}{c^2}\right)}\frac{1}{\gamma} \qquad (65)$$

$$v_z^{\prime} = \frac{dz^{\prime}}{dt^{\prime}} = \frac{dz}{dt^{\prime}} = \frac{v_z}{\left(1 + \frac{uv_x}{c^2} - \frac{u^2}{c^2}\right)}\frac{1}{\gamma} \qquad (66)$$

8 Discussion

What is the meaning of the equations (22), (48) and (49)? It may seem that there is no meaning in them, but if equations (4), (9) and (10) have a physical meaning, it should imply that the inverses of these equations have some meaning as well. Actually, these new equations are helpful in answering the following question: Why can only a massless particle spread in an infinite range? To answer this question, and to give these new equations some meaning, we have to introduce a new idea.

I will assume that there is a field (let us call it the 'k field') that it fills all the empty space throughout the entire universe. Elementary particles acquire their mass through their velocity with respect to that field. Suppose there is a field that has velocity equal to the speed of light relative to a rest observer. All elementary particles, which have zero velocity, must move with this field. In this case, the particle acquires what we called rest mass from the velocity of the field. Thus, we will introduce a relation between the mass and the velocity:

$$\Delta m \propto \Delta v$$

$$\Delta m = k \Delta v \qquad (67)$$

where k is the constant of proportionality. Now, if the particle moves with a velocity close to the speed of light ($v \sim c$) in the direction of the field, not in the opposite direction of the field, we have to use equations (4), (9) and (10) with both the known Lorentz transformations and the known relativistic velocity addition law. However, when the particle moves in the opposite direction of the field, we have to use equations (22), (48) and (49), and (58), (60) for the Lorentz transformations, and we have to use (64), (65) and (66) for the relativistic velocity addition law.

From the field assumption with the new inverse equations of relativity, we can answer the following questions:

1- Why can only massless particles spread in an infinite range?
2- Why did the experiments that measured the photon's mass produce different values?
3- What is the origin of elementary particles' masses?

These three questions are actually one question. For the first question, the answer is in equations (22) and (49). When the particle moves in the opposite direction of the k field, its mass must be reduced, according to equation (22), and its length will increase, according to the equation (49). When the particle's velocity reaches the speed of light in the opposite direction of the k field, its mass will be zero, and its length will be infinity:

$$m(v_{resist}) = m(c_{resist}) = 0 \qquad (68)$$

$$x' = \infty \qquad (69)$$

where v_{resist} is the velocity of the particle in the opposite direction of the k field, i.e., the particle resists the motion of the k field. Thus, if we assume that the photons have zero mass, their range will be infinity, based on the above equations. However, as we have stated previously, the photon has a nonzero mass, according to recent experiments. Equation (49) does not tend to infinity when $v < c$, but it becomes a very large quantity. In addition, equation (22) will be a very small quantity. These equations give a clear interpretation of the reason for the decrease in the electromagnetic field's range when the photon has a very small but nonzero mass. This phenomenon occurs when the photon receives a push from the k field. When it receives this push, its velocity decreases; this new velocity is just a little less than the speed of light. Thus, this phenomenon answers question two. For the third question, equation (10) predicts that the mass of particles are increases with velocity when the particle moves in the direction of the k field, and equation (22) predicts that the mass of particles are decreases when the particle moves in the opposite direction of the k field, It follows that the mass essentially depends on the motion. What we called the rest mass is determined by equation (67). When the particle does not move toward or opposite the direction of the k field, if it just moves with the field, similar to a boat moving with the current, equation (62) will determine its rest mass. However, in this case, all elementary particles must have the same value for their rest masses. Let us take the value of the electron's rest mass to be the smallest rest mass. From equation (62), the constant of proportionality will be:

$$k = \frac{m_{elec}}{v_{field}} = \frac{9.10938188 \times 10^{-31} kg}{3 \times 10^{8}\ m/s}$$

$$= 3.03646 \times 10^{-39}\ kg.s/m$$

Because equations (22) and (49) do not tend to zero and infinity only if the resisting motion equals the speed of light, the speed of the k field must be equal to the speed of light.

In that case, we can say that all particles in the universe are located between:

$$c^{+} < v < c^{-}$$

where c^{+} is the speed of light in the direction of k field and c^{-} is the speed of light in the opposite direction of the k field. No particle can reach the speed of light in the direction of the k field's motion, and also no particle can reach the speed of light in the opposite direction of the k field. If one can exceed the speed of light in the direction of the k field, one will see the past. In addition, if one exceeds the speed of light in the opposite direction of the k field, one will see the future.

Table (1-1) shows a comparison between the relativistic equations of motion with the inverse relativistic equations of motion, and table (1-2) shows the Lorentz transformations and velocity addition law for both relativistic and inverse relativistic motions.

Comparison	In high velocity, motion starts in the direction of the k field	In high velocity, motion starts in the opposite direction of the k

		field
Relativistic time interval	$\Delta t' = \gamma \Delta t$	$\Delta t = \frac{1}{\gamma}\Delta t$
Relativistic length	$x' = \frac{1}{\gamma}x$	$x' = \gamma x$
Relativistic mass	$m(v) = \gamma m$	$m(v) = \frac{1}{\gamma}m$

Table (1-1) A comparison between the relativistic equations and the inverse relativistic equations of time interval, length and mass.

TheLorentz transformation and relativistic velocity addition law for high velocity, where motion starts in the direction of the k field	The Lorentz transformation and relativistic velocity addition law for high energy, where motion starts in the opposite direction of the k field
$x' = \gamma(x - ut)$	$x' = \frac{1}{\gamma}(x - ut)$
$y' = y$	$y' = y$
$z' = z$	$z' = z$
$t' = \gamma(t - \frac{ux}{c^2})$	$t' = \gamma(t + \frac{ux}{c^2} - \frac{u^2 t}{c^2})$
$v_x' = \frac{v_x - u}{1 - \frac{uv_x}{c^2}}$	$v_x' = \frac{v_x - u}{(1 + \frac{uv_x}{c^2} - \frac{u^2}{c^2})}\frac{1}{\gamma^2}$
$v_y' = \frac{v_y}{1 - uv_x/c^2}\frac{1}{\gamma}$	$v_y' = \frac{v_y}{(1 + \frac{uv_x}{c^2} - \frac{u^2}{c^2})}\frac{1}{\gamma}$
$v_z' = \frac{v_z}{1 - uv_x/c^2}\frac{1}{\gamma}$	$v_z' = \frac{v_z}{(1 + \frac{uv_x}{c^2} - \frac{u^2}{c^2})}\frac{1}{\gamma}$

Table (1-2) A comparison between the relativistic equations and the inverse relativistic equations of the Lorentz transformations and the relativistic velocity addition law for high velocities, both in the direction of the k field and in the opposite direction of the k field.

9 The meaning of mass

Suppose, in a hypothetical scenario, that the concept of rest mass does not exist. Everything is in a state of motion, with a velocity close to the speed of light. In addition, suppose that we have a device capable of measuring the apparent value of particle's mass without knowing the value of its speed. In this case, the measured values are always relative. However, we will call this value the rest mass because we do not need the relativistic definition, which depends on the velocity. As a result, we have discovered something that depends on the velocity, and we also discovered the relation between this new mass and the old one by equation (10). Next, we find that we need to distinguish between these two masses. Therefore, we called one the rest mass and the other the relativistic mass. The relation between them is just the velocity.

I think that what happening now with the new ideas surrounding the mass of photons is similar to the above scenario. The measured mass of the photon has been considered to be its rest mass. However, because the photon is always in a state of absolute motion, the measured mass is the instantaneous mass, which is a result of the motion of the photon in the opposite direction of the k field. Therefore, in this case, the photon's measured mass is $m\ (v)$ in equation (22), and the value of the rest mass is derived from equation (67), as we discussed before. If we have a device that can measure the apparent values of the mass and the velocity at the same time, in this case, we can know the value of the rest mass from equation (10) or (22).

10 Conclusion

Einstein's thought experiment proved that the mass of a particle is a measure of its energy content. In addition, if the energy varies with velocity, the mass (the measured energy content) must also vary with velocity. Therefore, we have to distinguish between the rest mass and the relativistic mass. In section 3, we concluded that there is another way to prove that mass depends on the velocity: if the photon has a mass, as the recent experiments indicate, this mass must depend on the velocity, as we proved using equation (22). This equation is another version of the de Broglie relation for the relativistic case, i.e., it is another version of equation (20). However, equation (22) is similar to equation (10), so it must have the same physical meaning. Equation (22) indicates that the mass of the particle will decreasing with velocity, and it becomes zero when the particle reaches the speed of light. This equation implies that there must be another relation between proper time and relativistic time, which is given by equation (48). Because the length contraction is consistent with the time dilation, an equation that is consistent with equation (48) must take the form given by equation (49). To interpret these new equations, we have assumed the existence of a field, namely, the k field. From this field, we can conclude the following:

1- The speed of this field must equal the speed of light.
2- The light is moving in the opposite direction of the k field; therefore, it will appear extending at a speed equal to the speed of light in spacetime. The photons appear massless because they are moving

at the speed of light in the opposite direction of the k field; therefore, when the photons have a mass, these photons must be moving at a speed less than the speed of light, thus obeying equation (22).

3- When a photon has a mass, its range will be less than infinity, according to equation (49).

4- All elementary particles must have the same value of mass when they are moving in the k field without forward or backward motion in that field.

Notes

* We will analyse in detail the relation between energy, mass and relativistic mass in section 3.

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